Formal Analysis of Yoneda-Quillen Exact Categories

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Yoneda-Quillen exact categories provide a self-dual context in which the classical diagram lemmas of homological algebra may be established (see [1,2]). In [3], forms (which are faithful, amnestic functors) are used to define the noetherian form which is a self-dual context (different from that of Yoneda-Quillen exact categories) which covers all group-like structures. Furthermore, it was shown in [4] that the classical diagram lemmas of homological algebra may be established in the context of noetherian forms. In this talk, we will show that Yoneda-Quillen exact categories may be characterized using forms. We will also show that Yoneda-Quillen exact categories may be generalized to non-additive categories in such a way that retains the validity of homological diagram lemmas. While the generalized context is not self-dual, it covers not only Yoneda-Quillen exact categories, but also Borceux-Bourn homological categories (and hence all group-like structures, including topological groups), as well as the category of monoids with an exactness structure given by special Schreier extensions of monoids. All of these three contexts seem to fall out of the reach of noetherian forms.

References:

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